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THE INTERACTION OF AN ANTENNA WITH A PLASMA AND THE THEORY OF RESONANCE PROBES

by

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ABSTRACT

The impedance and the radiation field of a short antenna, which consists of two spherical conductors excited through very thin wires in phase opposition, is calculated. In the calculations the pressure tensor is replaced by a scalar pressure. A discontinuous model of the ion sheath is used.

The losses due to the radiation of electromagnetic and electron-acoustic waves are calculated and are expressed in terms of equivalent series resistances. The operation of resonance probes is discussed. It is shown that their resonant frequency is well below the electron plasma frequency if the probe radius is much larger than the Debye length. The significance of this result to both past and future ionospheric rocket probe experiments is pointed out. The limitations of the present treatment are discussed.

The Interaction of an Antenna with a Plasma and
the Theory of Resonance Probes

1. Introduction

It is customary in the theory of antennas embedded in a plasma to treat the plasma as a dielectric. In its most general form the dielectric constant of the plasma would be a tensor with complex elements; in the absence of an external magnetic field a scalar dielectric constant would be sufficient. This type of approach is known to be, at its best, only an approximation (c.f., for example Salpeter and Makinson 1949). The correct procedure would be the solution of a combination of the Boltzmann equation with Maxwell's equations. Unfortunately this path is beset with almost insuparable difficulties.

A compromise offers itself in the form of the so-called hydrodynamic approximation in which the pressure tensor of a plasma is replaced by a scalar pressure (Spitzer 1962; p.24). In this paper the problem of a spherical antenna (or more correctly two spheres excited in phase opposition through very thin wires) is treated by the application of the hydrodynamic approximation to a relatively crude model of the plasma sheath. Although this type of treatment neglects such effects as Landau damping, it is believed to describe most of the essential aspects of the interaction between the antenna and the plasma. For other applications of the same type of treatment the reader is referred to papers by Gould (1959), Fejer (1963), and Nickel, Parker, and Gould (1963). The present analysis and its conclusions are entirely different from those of Whale (1963) who also treated the excitation of electron-acoustic waves by introducing the concept of an isotropic pressure. Whale assumed the existence of interaction between the fluctuating quasi-electrostatic field and the electron-acoustic waves throughout the uniform plasma. In the present treatment the only interaction is taken to occur at the inner boundary (in reality in the sheath). Within the uniform plasma the electron-acoustic wave and the electromagnetic wave (or in the present limit the quasi-electrostatic field) propagate independently (c.f., for example Ginzburg 1961) and therefore cannot interact in terms of the hydrodynamic approximation.

2. The Excitation of Electron-Acoustic Waves

It is assumed that the unperturbed electron concentration is zero for $r < R$ and N for $r > R$. The distance r is measured from the origin of a coordinate system. This is admittedly a rather artificial model of the unperturbed ion sheath. It effectively assumes an abrupt potential barrier (this could be visualized as a hypothetical double-layer formed from infinitely heavy positive and negative ions) at $r = R$ which prevents the penetration of electrons inside the sphere at $r = R$. It is clear that the radial component of the mean electron velocity must vanish at $r = R$. Immediately inside the sheath there is assumed to be a spherical conductor whose (quasi-electrostatic) perturbation potential is taken to be a harmonic function of the time. (In principle the radius of the conductor could be taken as smaller than R without essentially modifying the analysis; this will not be done here.) Since there is no mean radial motion of the electrons at $r = R$, there is no fluctuating charged surface layer there and therefore both the perturbation potential and its normal derivative must be continuous at r = R. Inside the plasma the equation

$$mN\,\partial \underset{\sim}{v}/\partial t = Ne\,\underset{\sim}{\nabla} V - \gamma K T\,\underset{\sim}{\nabla} n \tag{1}$$

is taken to be valid where m is the mass, e the absolute value of the charge, N the unperturbed number density and n the perturbation in the number density of electrons (factors $e^{i\omega t}$ are taken for granted in n and in V) K is Boltzmann's constant, T is the temperature, and where the ratio γ of the specific heats is taken as 3 (Spitzer 1962). A combination of the divergence of equation (1) with the equation of continuity (satisfied by the velocity v of the electron gas)

$$\underset{\sim}{\nabla}.(N\underset{\sim}{v}) = -\,\partial n/\partial t \tag{2}$$

leads to

$$\frac{\partial^2 n}{\partial t^2} + \frac{Ne}{m}\underset{\sim}{\nabla}^2 V - \frac{\gamma K T}{m}\underset{\sim}{\nabla}^2 n = 0 \tag{3}$$

A combination of equation (3) and Poisson's equation in spherical coordinates and with spherical symmetry:

$$\underset{\sim}{\nabla}^2 V = \frac{d^2 V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = \frac{e}{\varepsilon_0} n \tag{4}$$

leads to

$$\underset{\sim}{\nabla}^2 n = \frac{d^2 n}{dr^2} + \frac{2}{r}\frac{dn}{dr} = \alpha^2 n \tag{5}$$

where $\alpha^2 = (\omega_N^2 - \omega^2)/u^2$, $\omega_N = (e^2 N/\varepsilon_0 m)^{\frac{1}{2}}$ is the electron plasma frequency and $u = (\gamma K T/m)^{\frac{1}{2}}$ is the velocity of electron-acoustic waves in the high frequency limit $(\omega >> \omega_N)$.

The general solution, which vanishes at infinity, of the differential equations (4) and (5) has the form

$$n = \frac{C_1}{r} e^{-\alpha r} \tag{6}$$

$$V = \frac{e}{\varepsilon_0 \alpha^2} n + \frac{C_2}{r} \tag{7}$$

where that part of the solution which is associated with the constant C_1, describes an electron-acoustic wave, whereas the part associated with C_2 describes a quasi-electrostatic field (the so-called induction field) which is a good approximation to the electromagnetic wave field within distances very much shorter than a wave length from the antenna.

The boundary condition v = 0 at r = R yield with the aid of equation (1)

$$\left(\omega_N^2 \frac{dV}{dr}\right)_{r=R} = \left(\frac{e u^2}{\varepsilon_0} \frac{dn}{dr}\right)_{r=R} \tag{8}$$

Substitution of n and V from equations (6) and (7) into equation (8) leads to the relation

$$C_2 = -\frac{e}{\varepsilon_0} \frac{u^2 \omega^2}{\omega_N^2(\omega_N^2 - \omega^2)} (1 + \alpha R) e^{-\alpha R} C_1 \tag{9}$$

Substitution of C_2 from equation (9) into equations (6) and (7) leads to the following relationship between the perturbation in the number density and the perturbation potential V at r = R:

$$\frac{n(R)}{V(R)} = \frac{\varepsilon_0 \omega_N^2}{e u^2} \left(1 - \frac{R \omega^2}{u(\omega_N^2 - \omega^2)^{\frac{1}{2}}}\right)^{-1} \tag{10}$$

Similarly, the effective, complex capacity of the conductor, defined here as the ratio of the charge on the conductor to the potential V(R) of the conductor is given by

$$C_{eff} = -\frac{4\pi R^2 \varepsilon_0 (dV/dr)_{r=R}}{V(R)} = 4\pi\varepsilon_0 R \frac{1 + R u^{-1}(\omega_N^2 - \omega^2)^{\frac{1}{2}}}{1 - R\omega^2 u^{-1}(\omega_N^2 - \omega^2)^{-\frac{1}{2}}} \tag{11}$$

Equations (6), (7), (9), (10), and (11) represent the main results of this paper and a discussion of their significance follows here. Equation (11) may also be used to calculate the impedance $Z_{eff} = (i\omega C_{eff})^{-1}$ of the antenna. The energy loss due to the radiation of electromagnetic waves has not as yet been included in the analysis; this will be done in a subsequent part of the present paper. The resistive part of the impedance Z_{eff} represents only the energy loss due to the "radiation" of electron-acoustic waves.

It is convenient to express the impedance Z_{eff} as a function of the parameters $Y = (4\pi\varepsilon_0 R\omega)^{-1}$ (the free space reactance of a sphere of radius R), $\delta = R u^{-1}\omega_N$ (the ratio of the radius R to $\gamma^{\frac{1}{2}}$ times the Debye length, and $\psi = \omega/\omega_N$ (the ratio of the frequency to the electron plasma frequency). The result is

$$Z_{eff} = -iY \frac{1 - \delta\psi^2(1-\psi^2)^{-\frac{1}{2}}}{1 + \delta(1-\psi^2)^{\frac{1}{2}}} \tag{12}$$

It is clear from equation (12) that Z_{eff} is pure imaginary for $\psi < 1$ or $\omega < \omega_N$. The absence of a loss term is explained in this case by the absence of propagating electron-acoustic waves. It may be seen from equation (12) that the impedance becomes infinitely high at the plasma frequency and that the impedance vanishes at the frequency where

$1-\delta\psi^2(1-\psi^2)^{-\frac{1}{2}}=0$. More will be said about the significance of this frequency later. At very low frequencies $Z_{eff} = -i\gamma(1+\delta)^{-1}$ and thus the impedance is smaller than the free space impedance by a factor $(1+\delta)^{-1}$. It is understandable that the effective capacity of the sphere is larger at low frequencies than the free space capacity because the alternating electric field does not extend to infinity but is confined to within the plasma sheath which in the case of large δ is much smaller than the radius. Figure 1 shows for $\psi<1$ the ratio of the imaginary part of Z_{eff} (the real part is zero for $\psi<1$) to the magnitude γ of the free space reactance as a function of the ratio $\psi=\omega/\omega_N$ of the frequency to the electron plasma frequency for different values of the ratio δ of the radius to λ Debye length. The curve for $\delta=\infty$ represents the well-known approximation in which thermal motions are neglected and the plasma is treated as a dielectric with an effective dielectric constant $\varepsilon_0(1-\omega_N^2/\omega^2)$.

For $\psi>1$ it is convenient to write equation (12) in the form

$$Z_{eff}=\frac{\gamma\delta(\psi^2-1)^{-\frac{1}{2}}}{1+\delta^2(\psi^2-1)}-i\,\frac{\gamma(1+\delta^2\psi^2)}{1+\delta^2(\psi^2-1)} \tag{13}$$

Figure 1 also shows the real and the imaginary parts of Z_{eff} for (in units of γ) as a function of ψ, for different values of δ. The case of $\delta=\infty$ again represents the approximation in which thermal motions are neglected and the plasma is regarded as a dielectric. The curves of Fig. 1 show the presence of a resistive component of the impedance for finite values of δ. The resistive, ohmical component represents the energy loss caused by the generation of electron-acoustic waves.

3. The Radiation of Electromagnetic Waves by Two Spherical Conductors Excited in Antiphase

The analysis of this paper can be extended to include radiation losses due to electromagnetic waves. The analysis of electromagnetic radiation by spheres oscillating in antiphase is of course somewhat artificial because in practice antennas resemble cylindrical conductors, rather than two spherical conductors excited in antiphase through thin wires whose capacity is neglected here. It is, however, relatively simple to extend the present analysis to cylindrical conductors, at least in an approximate manner; this extension whose results may be expressed in terms of Bessel functions, is not carried out in the present paper. The analysis of spheres oscillating in antiphase is considerably simpler and it illustrates the nature of the problem rather well.

If the distance D of the two spheres is much smaller than the electromagnetic wave length (and at the same time $R \ll D$) then the quasi-electrostatic field at a distance $r \gg D$ on the line connecting the spheres is $2C_2D/r^3$, where C_2/r is that part of the potential given by equation (7) which could be regarded as an approximation to the radiation field of a spherical radiator at distances much smaller than the wave length. At distances much longer than a wave length and in the plane that perpendicularly bisects the line connecting the two spheres, the magnitude E of the radiation electric field of the dipole antenna is then given by

$$E = C_2 D k^2/r = (C_2 D/r)\left[(\omega^2 - \omega_N^2)/c^2\right] \tag{14}$$

The radiation magnetic field H is given by

$$H = (\varepsilon_o C_2 D/r)\left[(\omega^2 - \omega_N^2)^{3/2}/c\omega\right] \tag{15}$$

where k is the wave number and c is the velocity of electromagnetic waves in vacuum. Equations (15) and (16) were derived by fitting the quasi-electrostatic dipole field to the radiation field of a dipole (Stratton 1941) in a medium with a dielectric constant $\varepsilon_o(1 - \omega_N^2/\omega^2)$. Equations (6), (7), and (9) can then be used to express C_2 in terms of V(R), the alternating potential of one of the spheres or in terms of the current supplied to the spherical conductors $I = i\omega C_{eff} V(R)$ where C_{eff} is given by (11). The Poynting vector $\underset{\sim}{E} \times \underset{\sim}{H}$ may then be expressed in terms of I, and the total radiated power P may be found by integration over a very large sphere. The result is

$$P = (6\pi\varepsilon_o)^{-1} D^2 \omega c^{-3} (\omega^2 - \omega_N^2)^{\frac{1}{2}} I^2 \tag{16}$$

and therefore the resistive component of the antenna impedance (the radiation resistance) due to the radiation of electromagnetic waves is

$$\rho_m = (6\pi\varepsilon_o)^{-1} D^2 \omega c^{-3} (\omega^2 - \omega_N^2)^{\frac{1}{2}} \tag{17}$$

The resistive component due to electron-acoustic waves is given by equation (14) as

$$\rho_a = 2Y \frac{\delta(\psi^2 - 1)^{-\frac{1}{2}}}{1 + \delta^2(\psi^2 - 1)} \tag{18}$$

Equations (18) and (19) show that ρ_m is zero at the electron plasma frequency and increases rapidly with frequency whereas ρ_α is infinitely high at the plasma frequency and decreases rapidly with frequency.

It is interesting to note that the expression (17) for ρ_m is the same as the expression for the radiation resistance of a short antenna of length D (whose capacity is entirely at its extremities) situated in a medium with a dielectric constant $\varepsilon_0(1-\omega_N^2/\omega^2)$. This is a significant result in view of the fact that in the calculation of the series reactive part of the antenna impedance the plasma can not be replaced by a medium with a dielectric constant $\varepsilon_0(1-\omega_N^2/\omega^2)$.

4. Application to Resonance Probes

The results of the previous section may be used to draw some tentative but very important conclusions about the behavior of resonance probes (Miyazaki et al, 1960). In such probes the change in the collected direct current, caused by the application of a radio frequency voltage is measured as a function of the radio frequency. It has been usually accepted (Miyazaki et al, 1960) as an experimental fact that the change in the collected current shows a resonant increase at the plasma frequency.

In this section the point of view is taken that the change in the collected direct current is due to rectification cause by the non-linear characteristic of a Langmuir probe. The amplitude of the radio frequency variations in the collected current will be proportional to the fluctuations n(R) in the number density (in a more accurate treatment the fluctuation in temperature would also have to be taken into account but these would in any case be proportional to the fluctuations in number density) and therefore equation (10) should really give the characteristics of a resonance probe. Using the parameters δ and ψ equation (10) may be written in the form

$$n(R) = V(R)\frac{\varepsilon_0 \omega_N}{e\,u^2}\left(1 - \frac{\delta\psi^2}{(1-\psi^2)^{\frac{1}{2}}}\right)^{-1} \tag{19}$$

Equation (19) shows that n(R) becomes infinitely large at the frequency for which

$$\psi = \left\{2^{-1}\delta^{-2}\left[-1 + (1+4\delta^2)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}} \tag{20}$$

This shows that resonance occurs near the electron plasma frequency ($\psi \sim 1$) only when $\delta \ll 1$; for large values of δ the resonance occurs well below the plasma frequency. Fig. 2 shows $(e\,u^2/\omega_N^2\varepsilon_0)\,|n(R)/V(R)|$ as a function of ψ for two values of δ. Fig. 2 illustrates that resonance occurs well below the plasma frequency when the probe radius is much larger than the Debye length. The resonant frequency given by (20) is the same as the previously discussed frequency where the impedance of the probe vanishes.

The present analysis clearly leads to the conclusion that the resonant response of the probe does not occur at the plasma frequency, as laboratory and space experiments are alleged to show, but always below the plasma frequency. Large errors could thus occur in rocket investigations of electron concentration in which the resonant frequency was identified with the plasma frequency, especially if the radius of the probe were much larger than the Debye length. Fortunately a relatively small sphere (R = 1 cm) was used in the ionospheric experiments (Aono et al, 1962) reported so far.

A discussion of the physical nature of this resonance sheds some light on the reasons for the occurrence of the resonance well below the plasma frequency. It is clear from equation (7) that the electric field of the probe consists of two parts. One is simply a quasi-electrostatic field which at short distances is the approximate form of the radiation field for frequencies above the plasma frequency while the other is the field associated with an electron-acoustic wave which is evanescent at frequencies below the plasma frequency.

At very low frequencies the quasi-electrostatic field becomes very small compared to the field of the evanescent electron-acoustic wave. This means that the alternating charge on the conducting sphere is perfectly shielded by a suitable (continuous) modification of the sheath. As the frequency is increased, the shielding becomes less perfect and a potential C_2/r appears outside the sheath. The present theory shows that the outside field opposes in phase the field within the sheath. With increasing frequency a situation is reached eventually where the potential drop outside the sheath just balances the potential drop inside so that no exciting voltage is required on the conductor; this is the condition for resonance.

It is clear that with the same shielding factor γ (defined as the ratio of the charge in the sheath to the charge on the conductor) the ratio ρ of the potential drop outside to the potential drop inside the sheath will increase as the ratio of the sphere radius R to the sheath thickness τ increases. The relation $\rho = (1-\nu)R\,\tau$ may be shown to apply if all the shielding charges are assumed to be on the sphere $r = R + t$. This is the physical explanation for the dependence of the ratio of the resonant frequency to the plasma frequency on the ratio of the Debye length to the probe radius.

An alternative, simpler but less quantitative, explanation is that the resonant frequency of the probe is lowered by a tight coupling to the medium. For a very small probe which is weakly coupled to the medium, the resonant frequency will be nearly equal to the plasma frequency. A larger probe has a lower resonant frequency because it is more tightly coupled to the plasma.

5. Conclusions

The simple analysis of this paper leads to certain interesting and significant results about the behavior of antennas in plasmas and about the interpretation of observations with the aid of resonance probes in plasmas. It is shown that resonance does not take place at the plasma frequency and that previous measurements made by the resonant probe method may have to be reinterpreted. In principle both the concentration and the electron temperature could be determined by simultaneous measurements of the resonant frequencies of two resonant probes of different size. The same information could be obtained from a single probe if an additional measurement (such as the additional direct current at very low frequencies) was made besides the determination of the resonant frequency. Impedance measurements (not necessarily above the plasma frequency) could also be used to determine the electron concentration and the electron temperature. It is to be remembered, however, that the results of this paper must be regarded as merely semi-quantitative in view of the oversimplified nature of the underlying assumptions.

The present treatment could, in principle, be further refined by using a more sophisticated model of the sheath even if the use of a scalar pressure term were retained. The treatment could also be applied, in principle, to conductors which have other shapes than spherical. The most severe shortcoming of the present approach is undoubtedly the use of a scalar pressure term and it is to be hoped that a calculation with the aid of the collisionless Boltzmann equation will be attempted in the future. It is also to be hoped that the conclusions of the present paper will be submitted soon to an experimental check.

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CAPTIONS

Fig. 1. The ratios of the real (interrupted line) and imaginary (solid line) parts of the impedance to the magnitude of the free space reactance as functions of the ratio ψ of the radio frequency to the electron plasma frequency.

Fig. 2. The dimensionless quantity $(e u^2/\omega_N^2 \varepsilon_0)|n(R)/V(R)|$ which is proportional to the number density fluctuations $n(R)$ produced by a given RF voltage $V(R)$, as a function of the ratio ψ of the radio frequency to the electron plasma frequency.

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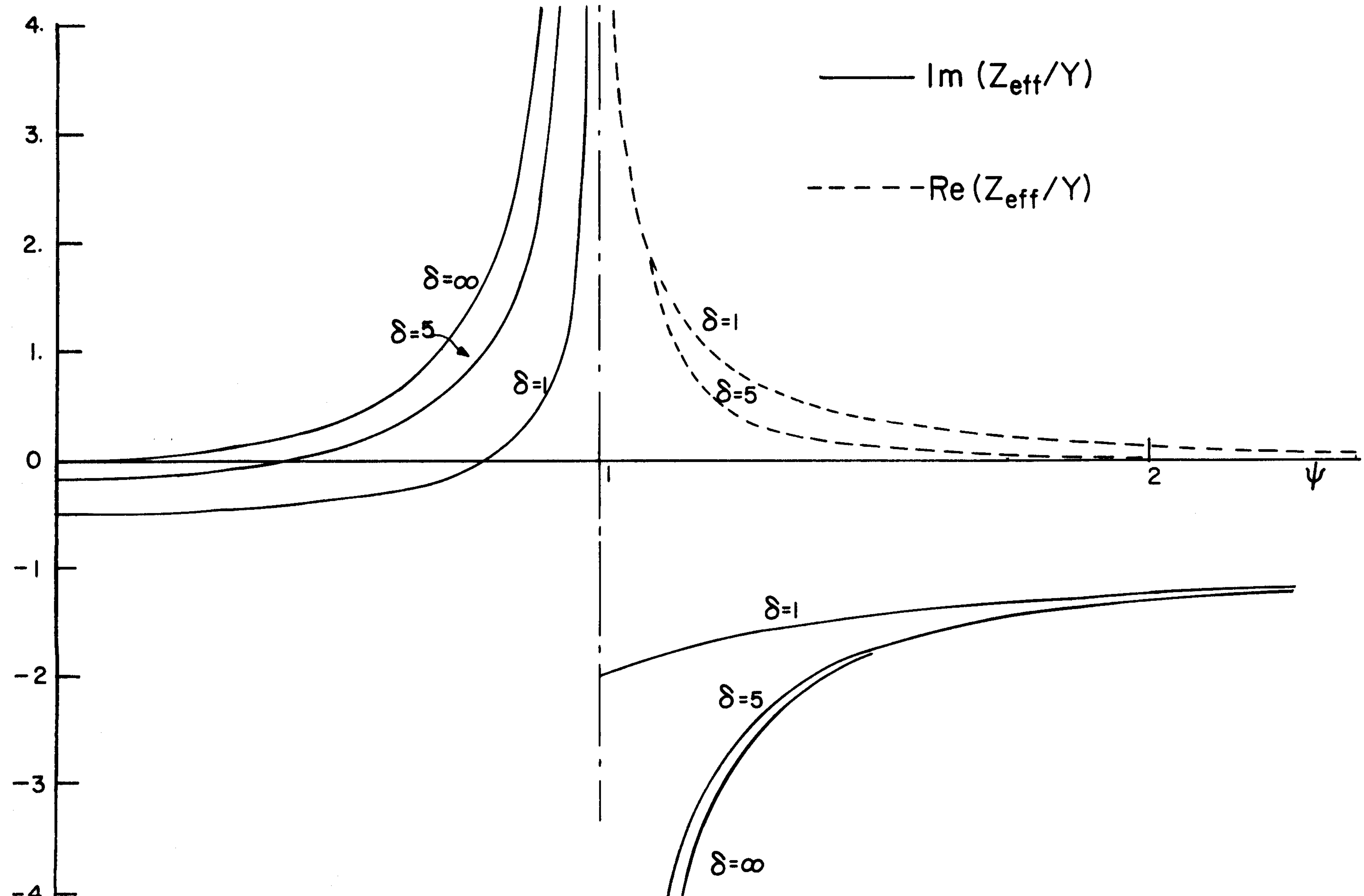

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Re (Z_eff/Y)
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3.
2.
1.
0
-1
-2
-3
-4
1
2
ψ
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δ=5
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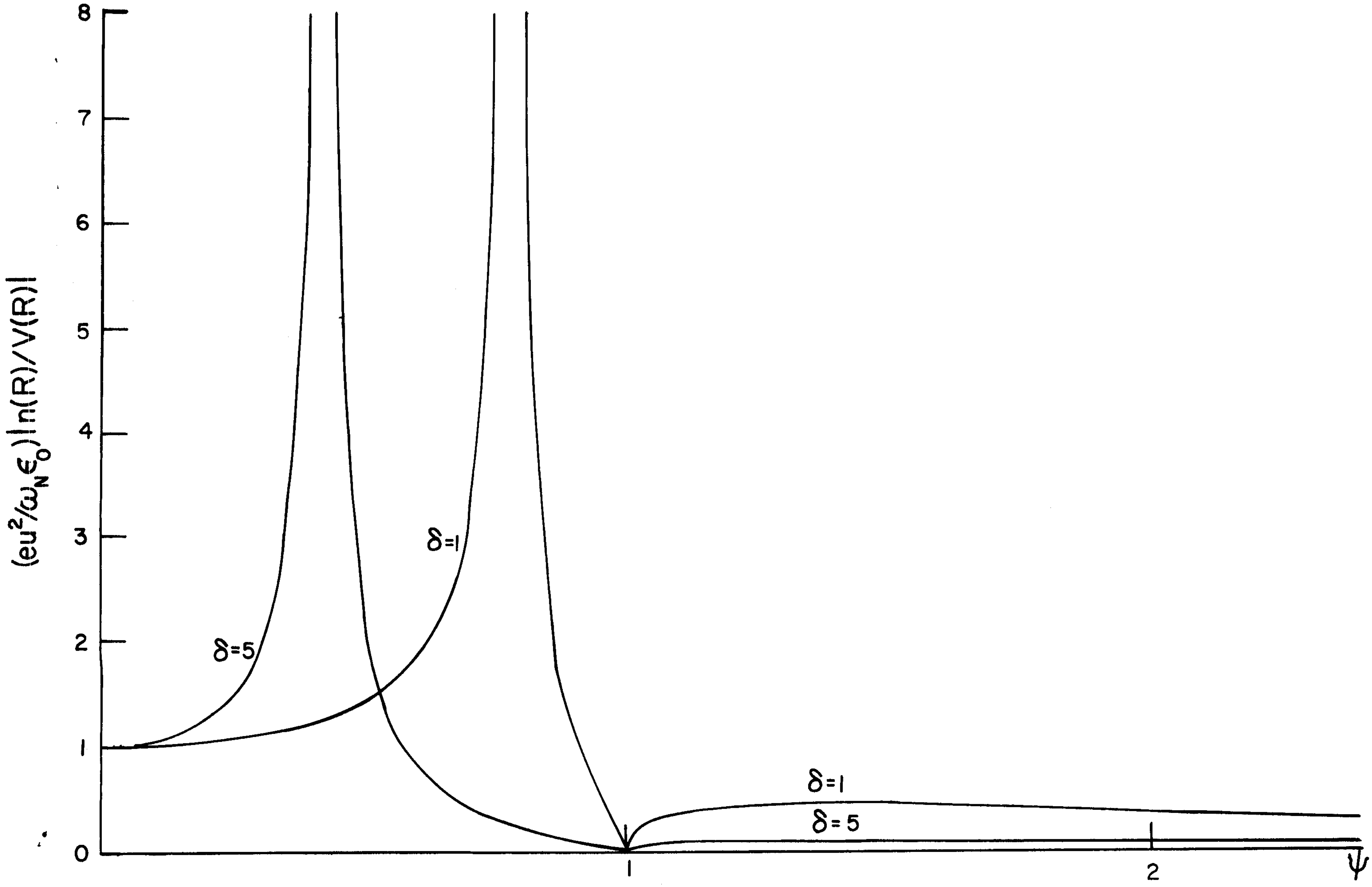

$(eu^2/\omega_N \epsilon_0)|n(R)/V(R)|$
0
1
2
3
4
5
6
7
8
δ=5
δ=1
δ=1
δ=5
1
2
ψ